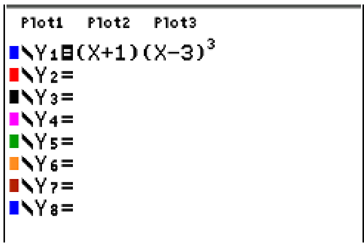
Chapter 4 / **Example 30**
Finding turning points and points of inflexion

Find any turning points and points of inflexion of $y=(x+1)(x-3)^3$ and justify your answers. Confirm your answers graphically.

Press [f1] $\boxed{\text{y=}}$ to display the equation entry screen.

Type $(x+1)(x-3)^3$ and press $\boxed{\text{enter}}$ to enter the equation as Y_1.

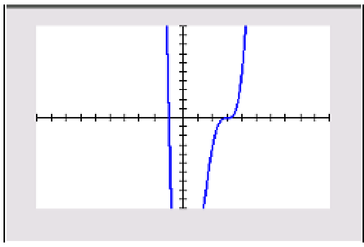


Press [f5] $\boxed{\text{graph}}$ to display the graph screen.

The GDC now displays the function:

$Y_1=(x+1)(x-3)^3$ with the default axes.

The default axes are $-10 \le x \le 10$ and $-10 \le y \le 10$.



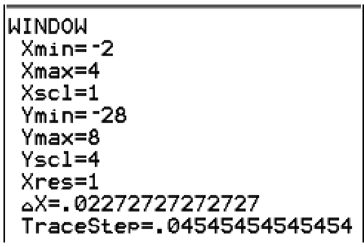
Change the window settings for a better view.

Press [f2] $\boxed{\text{window}}$[format]

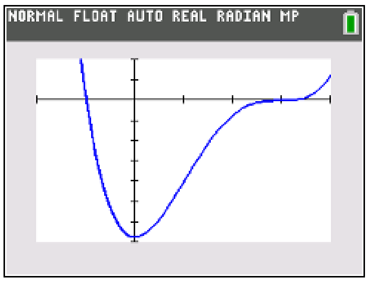
Set the axes to show $-2 \le x \le 4$ with a scale of 1 and $-28 \le y \le 8$ with a scale of 4.

You can leave the other items as they are.

Press [f5] $\boxed{\text{graph}}$ when you have finished.



The GDC displays the graph in a suitable window.



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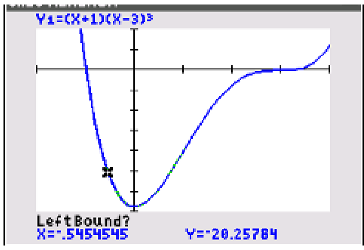
To find the minimum press $\boxed{\text{2nd}}$ [f4] [calc] 3:minimum.

You will need to give the left and right bounds of the region that includes the minimum.

The GDC shows a point on the curve and asks you to set the left bound. Move the point using $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleleft}$ and choose a position to the left of the turning point.

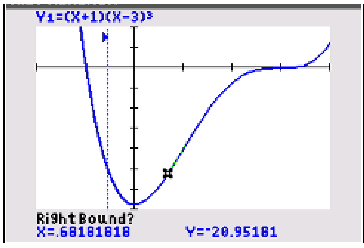
Press $\boxed{\text{enter}}$.



The GDC shows a line where you have set the left bound and a point on the curve.

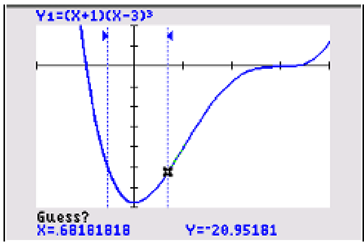
Move the point using $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleleft}$ and choose a position to the right of the turning point.

When the region contains the turning point, Press $\boxed{\text{enter}}$.



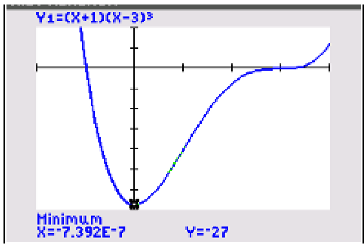
The GDC requires an initial guess for the position of the turning point. Choose the default position.

Press $\boxed{\text{enter}}$.



The GDC displays the minimum at $(0,-27)$.

Take care to interpret what the GDC displays. $X=-7.392\text{E}-7$ *means* $-7.392\times10^{-7}=0.0000007392$ *which is very close to zero. The small difference is due to the numerical way that the GDC calculates the value.*



Chapter 4 / **Example 30**

Finding turning points and points of inflexion

<table>
<tr>
<td>
Consider the nature of the function in the region around the point $(3,0)$.

Plot the first and second derivative functions.

Press [f1] [y=] to display the equation entry screen.

In Y_2 press [alpha] [f2] 3:nDeriv.

The template has spaces for the variable, x, the function and the value that it is evaluated at.

Enter X in the denominator and the function Y_1 using [alpha] [f4] 1:Y_1. Type X and press [enter].

In Y_3 enter the derivative of Y_2 in the same way.
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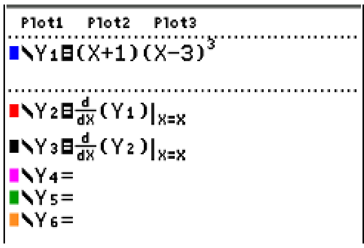

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<td>
Adjust the window to get a better view of the region around the point of the region around the point $(3,0)$.

Set the axes to show $-1 \le x \le 5$ and $-18 \le y \le 18$ with the same scales.
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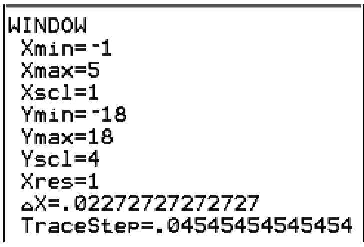

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Press [f5] [graph].

The GDC now displays the derivative function.

To find the horizontal inflexion of Y_1 find the minimum of Y_2.

At $(3,0)$ we see that $\frac{d^2y}{dx^2}$ changes from negative to positive.

This means that $\frac{dy}{dx}$ (the gradient of the curve) changes from decreasing to increasing (there is a minimum point), which means in turn that the concavity of the function changes from concave down to concave up.

The point is therefore a horizontal inflexion.
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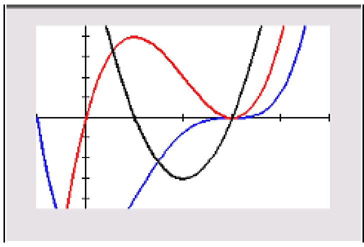
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